Chapter 21

Promoting Mathematics Achievement

Mark K. Harniss and Marcy Stein
University of Washington, Tacoma

Douglas Carnine
University of Oregon

INTRODUCTION

One of the most intense debates in the field of education currently is over mathematics instruction. Participation in this debate is not limited to educators, but includes many others interested in the mathematics performance of American students. The debate has drawn the attention of mathematicians, scientists, four Nobel laureates (Riley, 1999), as well as legislators and parents (Colvin, 1999). Questions of how best to teach mathematics have become so prevalent, in fact, that they are receiving attention in local, national, and international arenas. Student achievement in mathematics is discussed at Parent-Teacher Association (PTA) meetings, at universities, in state legislatures, and over the Internet. Furthermore, as countries of the world interact more, the relative effectiveness of American mathematics instruction, in comparison to instruction common in other countries, continues to be questioned (Bracey, 2000).

The debate has been fueled at least partially by the poor mathematics performance of American students. Students struggle to perform satisfactorily in math at all grade levels. According to the 1996 National Assessment of Educational Progress Mathematics Report Card, only 21% of fourth-grade students perform well enough in math to be labeled "Proficient" in the subject. This percentage drops to 16% for high school seniors (Reese, 1996). Ultimately, most American students graduate from high school without the mathematics competencies required for many career options.

In order to understand the seriousness of the problem in the United States, researchers have compared the mathematics performance of American students to that of students from other countries. The results of the Third International Mathematics and Science Study (TIMSS), a comprehensive educational survey based on participating students from 40 countries, revealed that significant performance gaps exist between

American students and students in other countries (e.g., gaps of 1.5 standard deviations between American students and students in Singapore) (Loveless & Diperna, (2000).

To help explain the TIMSS results, researchers examined the curricula used by mathematics teachers in various countries. First, TIMSS researchers found that American mathematics textbooks are *longer* than most and cover a greater number of topics. For example, a typical fourth-grade American mathematics textbook contains an average of 530 pages. In contrast, the average length of international mathematics textbooks is only 170 pages. One explanation for the differences is that most U.S. states have standards that guide the development and production of textbooks. Because the standards are not identical in each state, publishers are compelled to include the greatest number of topics to meet the greatest number of standards so that their textbooks will sell to the greatest number of teachers. However, because of the large number of topics included at any given grade level, textbooks often allocate the same amount of space for simple skills as they do for more complex skills. Moreover, the textbooks place little emphasis on systematic skill progression or skill mastery. According to the study, most American textbooks sacrifice depth for breadth (Valverde & Schmidt, 1998).

In addition to curricular issues regarding mathematics instruction, researchers are finding that many American teachers are simply underprepared to teach mathematics. According to Wu (1999), American teacher preparation courses are notably lacking in the area of mathematics instruction. This study also revealed that some American elementary teachers themselves had difficulties performing certain mathematical calculations, such as dividing fractions. Furthermore, none of the teachers in the study could adequately explain the mathematical reasoning in the algorithms of the problems they were given, nor could they devise real world applications or offer proofs. This problem has also been highlighted in Ma's (1999) comparison of Chinese and American educators.

Given that the curriculum materials teachers are given may not be adequately designed and that some teachers may not be adequately prepared to teach mathematics, especially to students with disabilities or those at risk for academic failure, we propose two options for the role of the school psychologist. First, at the school or district level, school psychologists can help teachers and administrators *identify evidenced-based models* of mathematics instruction that have documented academic success with diverse student populations. Depending on availability of resources, administrators may explore the possibility of implementing one of these evidenced-based schoolwide models at a school or throughout an entire district. Second, at the classroom level, school psychologists may be able to *provide support to individual teachers* by helping them modify their curriculum materials to best meet the needs of their low-performing students.

To assist school psychologists in pursuing either of the options mentioned above, we have divided this chapter into two major parts. First, we examine several models of mathematics instructional interventions that are based on a strong body of research. Next, we outline general principles of curriculum modification that are derived from examining the instruction in those models. We conclude with recommendations on how school psychologists can contribute to the mathematics achievement of diverse student populations.

MATHEMATICS INSTRUCTIONAL INTERVENTIONS

We have identified four different instructional intervention models, all of which have been supported by empirical research, to describe briefly in this section: The Missouri Mathematics Effectiveness Project, Cooperative Learning, Classwide Peer-Tutoring, and Direct Instruction. Each of these models was designed as a developmental (i.e., not remedial) model to meet the needs of students representing a wide range of abilities, especially those who may be at risk for academic failure. In fact, much of the research on each model was conducted with students from economically disadvantaged communities.

The Missouri Mathematics Effectiveness Project

This project is among the first experimental studies to demonstrate the relationship between specific teaching activities and student achievement in the area of mathematics (Good & Grouws, 1979). Previous to this study, the teacher effectiveness literature consisted largely of process-product studies that yielded correlations between specific teaching behaviors and student performance. In fact, Good and Grouws based their instructional intervention on the results of a naturalistic observation study of mathematics teachers from which they were able to identify a set of factors that discriminated the highly effective teachers from those who were less effective. After integrating their findings with variables suggested by other teacher effectiveness studies, they derived five key instructional features to include in their model of mathematics instruction: Daily Review, Development, Seatwork, Homework Assignment, and Special Reviews. (See Good, Grouws, & Ebmeier, 1983, for a more complete description of these variables.)

Good and Grouws then tested the effectiveness of providing staff development in the form of inservice training on this model by comparing the mathematics performance of students in the treatment classrooms to that of students in the control classrooms where teachers were encouraged to maintain their own teaching style. Importantly, they first determined that the level of implementation of their model was quite high. Only 2 of the 21 treatment teachers scored low on measures of implementation, and the treatment teachers did exhibit more of the treatment teaching behaviors than did the control teachers. The results of this study provided evidence that the Missouri Mathematics Effectiveness Model was effective in increasing mathematics achievement as measured by both standardized and criterion-referenced mathematics tests.

This study is significant for several reasons. First, the researchers demonstrated under experimental conditions that *how* students are taught mathematics has a significant impact on what students learn about mathematics. Perhaps even more significantly, the researchers demonstrated that teachers using this instructional system could increase the achievement of low-income, inner city students, and overcome the risk of low expectations. Another relevant finding from this study is that the "development" component of the instructional system (i.e., the explanations and initial teaching of new concepts) was the most difficult component for teachers to implement. The implications that this finding has for the evaluation, selection, and use of instructional material are discussed in the section on designing effective mathematics instruction.

Cooperative Learning

Cooperative learning is an instructional model characterized by the use of student teams to enhance learning. In his analysis of cooperative learning, Slavin (1983) found that the interdependence of two features, (a) group goals and (b) individual accountability, was critical for student success in any cooperative learning model. The Team-Assisted Instruction (TAI) program for mathematics instruction is an example of the application of cooperative learning to the teaching of mathematics (Slavin, Leavey, & Madden, 1984). In TAI, students work in heterogeneous teams but on mathematics materials designed to meet their individual needs. Members of the team help each other with problems as well as check work and perform other "managerial" duties. In addition to the team work, teachers work with homogeneous groups drawn from each team at specified times delivering direct instruction on important mathematics concepts and skills. At the end of each week, all team members are assessed using criterion-referenced measures and group points are allocated according to team performance. TAI has been described as integrating both individualized instruction and direct instruction with cooperative learning teams and cooperative incentives (Slavin & Karweit, 1984). It should be noted that other generic models of cooperative learning such as Student Teams-Achievement Division (STAD) also have been applied to the teaching of mathematics (Nattiv, 1994).

Several studies have demonstrated that students taught mathematics using TAI have outperformed students in control conditions (Slavin et al., 1984; Slavin & Karweit, 1984; Stevens & Slavin, 1995). A particularly interesting study, involving two experiments, investigated the effects of three different instructional models commonly used to teach mathematics: (a) individualized instruction as exemplified by TAI, (b) whole-class instruction, using the Missouri Mathematics Program (MMP), and (c) within-class ability grouping using the principles of the MMP applied to two homogeneous groups of students. In both experiments, TAI and the ability-grouped instruction increased achievement in mathematics computation more than the MMP and traditional whole class instruction. The latter was present in Experiment 2 only. No differences were found in both experiments for TAI and ability-grouped instruction.

In Experiment 2, students taught using the MMP significantly outperformed those who were taught using traditional instruction, thereby replicating earlier research on the MMP. No treatment effects were found in either experiment on measures of mathematics concepts and applications. Finally, in both experiments, students' attitudes toward math class were more positive for TAI groups. In addition, more teachers reportedly chose TAI when given the opportunity to choose any method, other than the one they had used before, for training and materials.

ClassWide Peer Tutoring

ClassWide Peer Tutoring (CWPT) is an instructional system that employs simultaneous tutoring throughout the entire class. The tutor-tutee pairing is established at the beginning of each week along with assignments of tutoring pairs to teams. All students are both tutors and tutees and as such all students are taught to provide feedback through appropriate error correction. CWPT is similar to TAI in its use of interdependent social reward structures for both individual and team performance. However, CWPT differs from TAI in that mathematics instruction and practice are organized for an entire class and not completely individualized. CWPT was designed to maximize academic engaged time in the classroom, promote high levels of mastery, and ensure sufficient content coverage. For more detail on CWPT, see Greenwood, Maheady, and Delquadri in this volume.

While CWPT is not restricted to use with economically disadvantaged students, this instructional model has been effective in producing achievement gains for these students commensurate with their more advantaged peers. In a longitudinal study of first-through fourth-grade students (Greenwood, Delquadri, & Hall, 1989), the experimental low socioeconomic status (SES) group using CWPT achieved greater gains in mathematics (and language and reading) than did the low SES control group. At the same time, no differences in academic performance were evident between the low SES experimental group, using CWPT, and a high SES comparison group, using traditional instruction. Notably, in this study, student satisfaction with tutoring was quite high.

In a follow-up study 2 years later at the end of sixth grade, Greenwood, Terry, Utley, Montagna, and Walker (1993) examined the achievement outcomes for the low SES experimental group, the low SES control group and the nonrisk group. They found that the CWPT students maintained their advantage over their low SES peers on the mathematics subtest of a standardized achievement test, produced significantly higher scores on tests of science and social studies not previously measured, and were referred less often for special education services and for less restrictive services. Findings from this study also demonstrated that the CWPT students performed similarly to the nonrisk students in the area of mathematics.

A substantial amount of research has been conducted on different models of classwide peer tutoring (Fantuzzo, Polite, & Grayson, 1990), the effects of supplemental peer practice (Good, Reys, Grouws, & Mulryan, 1989/1990; Kohler, Ezell, Hoel, & Strain, 1994), and teachers' perceptions of classwide peer tutoring and curriculum-based measurement (Phillips, L. S. Fuchs, & D. Fuchs, 1994). Findings from both short-term and longitudinal studies support the continued use of CWPT and suggest that this model is an exemplary preventive and prereferral intervention strategy for those students most at risk for academic failure.

Direct Instruction

The term *direct instruction* has appeared in the literature in various contexts, often referring to systematic instruction. The *Direct Instruction* model (DI), described in this section, refers to a comprehensive instructional model involving many of the features of the previously discussed models including (a) organizational structure, in this case, small group instruction; (b) systematic teacher preparation; (c) the use of guided and frequent practice with feedback; as well as (d) a system for monitoring student and teacher performance.

However, a defining feature of the Direct Instruction model is the use of a carefully designed curriculum based on the text *Theory of Instruction* (Engelmann & Carnine, 1991), that explicitly teaches not only algorithms for computation but also *generalizable rules and strategies* for solving problems. The nature of the DI curriculum differs from those used in other instructional models primarily in the specificity of the major instructional strategies. DI emphasizes the identification of, and instruction in, (a) background knowledge, (b) a carefully designed sequence of instruction, and (c) the use of cumulative introduction and review to integrate new and previously introduced skills and concepts (Carnine, Grossen, & Silbert, 1995; Gersten, Woodward, & Darch, 1986).

In 1968, the Direct Instruction Model was selected as one of the nine major instructional models to be evaluated as part of Project Follow Through, a federally funded project designed originally for the development and implementation of innovative teaching practices in schools serving low-income student populations. Over 180 school districts have been involved in the Follow Through Project. The major models it has used reflect a range of educational philosophies including Piagetian approaches and models based on discovery learning, in addition to three models based on behavioral principles. The Direct Instruction Model was implemented in communities throughout the United States including Flippin, Arkansas; the Rosebud Sioux Reservation in South Dakota; inner city schools in New York and Washington, D.C.; rural schools in Williamsburg County, North Carolina; as well as in Hispanic communities such as East Las Vegas, New Mexico.

ABT Associates conducted an independent evaluation of Follow Through for the U.S. Office of Education (Stebbins, St. Pierre, Proper, Anderson, & Cerva, 1977). The measures used to determine mathematics achievement were the math problem-solving and math computation subtests of the *Metropolitan Achievement Test* and the mathematics subtest of the *Wide Range Achievement Test*. Data from the Follow Through evaluation were quite revealing. The results indicated that low-income primary-grade students who received the full 3- to 4-year Direct Instruction mathematics program outperformed students who were taught using other approaches on all *Metropolitan Achievement Test* mathematics subtests. The Direct Instruction students achieved a level higher than was expected for students of similar demographic characteristics, a level that was commensurate with that of their middle-income peers (Gersten & Carnine, 1984).

Results from a follow-up study of fifth- and sixth-grade students who had been taught using DI mathematics programs in the primary grades but were no longer in the programs indicated consistent positive findings in the area of mathematics problem solving, weaker but significant effects in mathematics concepts, and null effects in computation (Becker & Gersten, 1982). The results from the follow-up study seem to suggest that the DI students maintained those skills that were most generalizable—for example, problem-solving skills. Once the students left third grade, they were taught multiplication and division computation skills by use of more traditional instruction and no longer demonstrated significant achievement in the area of computation. Notably, evidence on the effectiveness of Direct Instruction is not limited to data from the Follow Through program but comprises a rich literature that has been summarized recently by Adams and Engelmann (1996).

Summary

The four instructional models discussed in this chapter share many features including teacher-directed instruction, high levels of academic engagement, continuous progress monitoring, and frequent feedback to students. More importantly, they all have been evaluated experimentally to determine their relative effectiveness in improving the mathematics performance of students, especially those at risk for academic failure. Notably, most of the features of the instructional models outlined above are related to either what is taught or how it is being taught.

CURRICULUM MODIFICATION

Not all school psychologists will have the opportunity to implement the schoolwide programs previously described. As an alternative, school psychologists can provide support to individual teachers by helping them modify their existing mathematics curriculum. Three areas of a curriculum should be addressed in this modification: (a) instructional goals, (b) instructional strategies, and (c) formative assessment. These three areas are discussed in more detail in the following sections.

Instructional Goals

All teachers have learning goals for their students, but some goals may be more important than others. When evaluating mathematics instruction in the classroom, school psychologists can assist teachers in determining the *relative importance* of these goals. Relative importance is determined by examining two factors. First, learning goals should focus on skills that are most frequently used. Second, learning goals should address the foundational concepts or "big ideas" in mathematics.

The first factor in determining the importance of student learning goals is frequency of use. Given a limited amount of instructional time, teachers must select goals that address important concepts and skills. For example, teaching students to write numbers in the billions is less important than teaching a strategy for solving ratio and proportion word problems. One means of determining importance is by examining district grade-level expectations often found in curriculum guides.

A second and related means of determining importance is by ensuring that student learning goals focus on *big ideas*. Big ideas are major organizing principles that have rich explanatory and predictive power and are applicable in many situations and contexts

(Kame'enui & Carnine, 1998). Guidelines written by The National Council of Teachers of Mathematics (NCTM, 2000) suggest that, "Foundational ideas like place value, equivalence, proportionality, function, and rate of change should have a prominent place in the mathematics curriculum because they enable students to understand other mathematical ideas and connect ideas across different areas of mathematics" (p. 15).

Davis (1990) points out that too often teachers focus their instruction on "small ideas." For example, arbitrary procedures such as cross-multiplying to solve problems like x/a = b/c. Such procedures frequently rely only on rote recall, preempting the possibility that students *will infer* the important mathematical principles underlying them. In addition, many mathematics textbooks are not designed using the concept of big ideas. For example, in most textbooks students are expected to learn different formulas to calculate the volume of seven three-dimensional figures:

- Rectangular prism: l • w • h = v
- Wedge: 1/2 • l • w • h = v
- Triangular pyramid: 1/6 • l • w • h = v
- Cylinder: p • r^2 • h = v
- Rectangular pyramid: 1/3 • l • w • h = v
- Cone: 1/3 • p • r^2 • h = v
- Sphere: 4/3 • p • r^3 = v

These equations emphasize rote formulas rather than big ideas. An analysis based on big ideas reduces the number of formulas students must learn from seven to slight variations of a single formula, that is, the area of the base times the height (B • h). This approach enhances understanding while simultaneously reducing the quantity of content to be learned, remembered, and applied.

Big ideas are found throughout mathematics. For example, the four operations of addition, subtraction, multiplication, and division rest upon a limited set of big ideas. These ideas include place value; the distributive, commutative and associative principles; equivalence; and number sense (i.e., primarily the concept of composition and decomposition of numbers in a base 10 system [Ma, 1999]). Figure 1 provides a detailed description and examples of these big ideas for operations. These big ideas interweave throughout the teaching and learning of the operations. When they are clearly understood by teachers and students, they serve as the conceptual underpinnings for understanding the operations. (See Carnine, Dixon, & Silbert, 1998, for a lengthier discussion of big ideas in mathematics.)

FIGURE 1

Big Ideas in Operations

Big Idea	Example
Place value is the understanding that in our number system, the "place" a number holds in a sequence of numbers gives information about that number.	In the number 324, the 3 at the beginning of the number is a one hundreds number. We know that the placement of the 3 tells us that there are three units of 100, or three 100s, in that number. Similarly, the location of the 2 tells us that there are 2 units of 10 in the number.
Expanded notation is simply the awareness by learners that you can reduce a number to its constituent units.	The number 432 is composed of four 100s, three 10s, and two ones which can be represented in an equation as 100 + 100 + 100 + 100 + 10 + 10 +10 + 1 + 1 = 432 or conversely as 400 + 30 + 2 = 432.
Commutative property: The order in which numbers are placed in the equation can be changed without affecting the outcome. a + b = b + a	Addition and multiplication are commutative: In addition, 5 + 6 = 11 and 6 + 5 = 11 In multiplication, 4 x 5 = 20 and 5 x 4 = 20 Subtraction and division are not commutative: In subtraction, 6 – 1 = 5 and 1 – 6 = –5 In division, 8 ÷ 4 = 2 and 4 ÷ 8 = 0.5
Associative property: The groupings in which numbers are placed in the equation can be changed without affecting the outcome. (a + b) + c = a + (b + c)	Addition and multiplication are associative: In addition, (6 + 3) + 3 = 12 and 6 + (3 + 3) = 12 In multiplication, (3 x 2) x 5 = 30 and 3 x (2 x 5) = 30 Subtraction and division are not associative: In subtraction, (15 – 5) – 3 = 7 and 15 – (5 – 3) = 13 In division, (32 ÷ 8) ÷ 2 = 2 and 32 ÷ (8 ÷ 2) = 8
Distributive property: You can distribute numbers in a problem that includes multiplication and addition. a x (b + c) = (a x b) + (a x c) You can also distribute numbers in an equation that includes division and subtraction or addition. (a + b) ÷ c = (a ÷ c) + (b ÷ c)	 5 x (3 + 2) = (5 x 3) + (5 x 2) (8 + 4) ÷ 2 = (8 ÷ 2) + (4 ÷ 2)
Equivalence: The quantity to the left of the equal sign (=) is the same as the quantity to the right.	32 + 15 = 47 16 + 16 + 15 = 47 8 + 8 + 8 + 8 + (5 x 3) = 20 + 20 + 7 *Note:* Many students interpret the equal sign as an operation (e.g., "when I see the equal sign I add, subtract, etc.") rather than as a relationship (e.g., "when I see the equal sign I know that the quantity on one side must be the same as the other side").
The *"rate of composition/decomposition of numbers"* (Ma, 1999) is a form of number sense. The rate of composition (or decomposition) of sets of numbers in our base 10 system is simply 10.	When you have accumulated 10 ones you have one 10. When you have accumulated 10 tens you have one 100 and so on. This concept is sometimes referred to as unitizing, that is, creating a tens unit from ten ones. Similarly, when you remove a one from a 10 you have nine ones, that is you have decomposed the ten.

Instructional Strategies

School psychologists can help teachers evaluate the quality of the instructional strategies available to them in their mathematics textbooks by considering four factors: (a) clarity of instructional strategies, (b) prerequisite knowledge, (c) scaffolding, and (d) review and integration.

Clarity of instructional strategies. Any routine that leads to both the acquisition and utilization of knowledge can be considered a strategy (Prawat, 1989). Prawat recommends that efficient strategy interventions be of intermediate generality. That is, efficient strategies fall somewhere between the extremes of being narrow in application and consistently reliable and being broad but not necessarily reliable.

An example of a narrow strategy would be teaching students to "carry the one" when solving addition problems with renaming to the 10s column. Using this strategy, students may not recognize the role of place value in renaming and may apply the strategy inappropriately. At the other extreme, an instructional strategy may be so general that it is little more than a broad set of guidelines. For instance, a broad problem-solving strategy such as "draw a picture" or "read, analyze, plan, and solve" will not help students who do not already have the component skills necessary for solving the problem.

A major challenge of instruction is to identify or modify strategies to assist students who do not develop these strategies on their own. Once teachers identify these strategies, they need to *teach them explicitly*. The support for explicitly teaching strategies is quite strong (Cardelle-Elawar, 1995; Carnine & Stein, 1981; Charles, 1980; Gleason, Carnine, & Boriero, 1990 Leinhardt, 1987; Moore & Carnine, 1989; Resnick, Cauzinille-Marmeche, & Mathieu, 1987; Resnick & Omanson, 1987; Woodward, Baxter, & Robinson, 1999). It should be noted that some educators confuse explicit instruction with rote instruction. With explicit instruction students do not memorize the answers to a set of examples; they learn a *set of procedures* that when successfully applied to the examples will result in greater accuracy and deeper understanding.

Prerequisite knowledge. School psychologists and teachers also should be aware of the need to identify important background knowledge critical to the strategies being taught. If students lack this knowledge, teachers must provide appropriate background instruction. If students have been taught this knowledge previously, they may still need to be "primed." That is, students will need to be reminded of what they know and shown how and when their previous knowledge supports their learning of new knowledge. This explicit linking of old to new knowledge is critical in helping students develop rich conceptual networks of mathematical knowledge.

Instruction on prerequisite skills should ideally occur prior to the introduction of a strategy requiring those skills. An example of critical prerequisite knowledge can be seen with an example from instruction on fractions. Students must know how to find the least common multiple of a set of numbers before they can successfully solve addition problems with fractions having unlike denominators. Thus, teachers would be wise to ensure that students are competent in finding the least common multiple before moving into teaching addition with unlike denominators.

Scaffolding. Scaffolding is a means by which students receive support from teachers in various forms as they are learning new content or strategies. More support is provided during initial instruction with a reduction of support as students become more proficient. (See Kame'enui & Carnine, 1998, for more information on scaffolding.)

Scaffolding in mathematics can include teacher prompting through focused questions, graphic support (e.g., the use of grids to assist students in column alignment), and peer collaboration in problem solving. Eventually all forms of support should be unnecessary as students demonstrate content or skill mastery.

An important part of scaffolding a task appropriately is to determine students' prerequisite knowledge accurately and target the task toward their instructional level. Vygotsky (1978) used the term *zone of proximal development* to describe situations in which students' cognitive ability matches the cognitive requirements demanded by an instructional activity.

Review. Most teachers would agree that review is important. However, all review is not the same. Sometimes review can be punishing in its repetitiveness or the amount can be aversive. Other times, review can be disconnected or out of sync with student learning. Too often, review is used only immediately after learning a new concept and then is dropped before students firmly attain a skill or concept. Developing good cycles of review takes careful thought and attention to detail.

Research strongly supports certain types of review (e.g., Dempster, 1991). Teachers should plan review that is sufficient (i.e., adequate to initially learn the content information), distributed (i.e., integrated over time so students do not forget what they have learned), cumulative (i.e., built upon previously learned information), and varied (i.e., applied in a variety of contexts) (Kame'enui & Carnine, 1998).

When review is planned appropriately, students not only understand individual strategies, but also learn how different instructional strategies are related to one another (Nickerson, 1985; Prawat, 1989; Van Patten, Chao, & Reigeluth, 1986). When scheduling review, it is also critical that only important information be placed into the review cycle. If instructional content has been identified as important to teach, then that content is worth the time spent reviewing.

Formative assessment. Teachers need to understand clearly how they know if and when their students have learned mathematical content. Many teachers are not taught in their teacher preparation programs how to use objective measures to evaluate student learning over time for the purpose of deciding whether students are benefiting from instruction. Moreover, teachers often are not taught how to link assessment data to instructional decision making.

Traditionally, school psychologists are trained to use published norm-referenced tests to identify students with disabilities and subsequently make placement recommendations. However, school psychologists can be of great assistance to teachers in identifying more functional measures of student mathematics performance as well as in helping teachers use data on student performance to modify classroom instruction.

Two types of assessment are critical in mathematics: (a) criterion-referenced assessment with error analysis, and (b) formative evaluation of student progress over time.

Ideally, teachers should integrate *both* criterion-referenced measures and formative evaluation over time into their mathematics instruction. Because school psychologists are familiar with these types of assessment, we will spend limited time describing each type.

In short, criterion-referenced measures help teachers know whether the instructional strategies they taught allowed students to achieve mastery on the goals they have established. Most mathematics programs provide a wide variety of criterion-referenced (i.e., mastery-oriented) assessment options from which teachers can choose. School psychologists need to become familiar with the advantages and disadvantages of the available options in order to help teachers make decisions about the most useful type of assessment.

In addition, school psychologists might assist teachers in conducting error analyses. An error analysis of the mistakes made on a criterion-referenced assessment helps teachers identify specific content for which students need additional instruction to help move students toward mastery. When examining student errors in mathematics, teachers should note whether the error appeared to be caused by (a) lack of knowledge of basic facts, (b) lack of a reliable problem-solving strategy, (c) lack of prerequisite knowledge, or (d) lack of motivation. If a student consistently misses problems requiring knowledge of basic facts, remediation will need to include independent work on facts. (See Stein, Silbert, & Carnine, 1997, for recommendations about setting up a fact mastery program.) If students lack the appropriate strategy to solve a certain type of problem, teachers need to reteach that strategy to the students using scaffolded instruction until students can demonstrate mastery.

As we noted earlier, some students may lack prerequisite knowledge for mastery of a given strategy. For example, students must know how to add a single column of one-digit numbers (e.g., 4 + 3 + 2) if they are going to be successful in renaming in column addition (e.g., 24 + 13 + 42). As teachers conduct an error analysis, they must be able to determine whether the student errors were attributable to lack of knowledge of the appropriate strategy or lack of prerequisite knowledge.

Finally, teachers must be able to determine whether student errors are caused by a lack of motivation rather than a lack of knowledge. One way for teachers to explore motivation as a cause of poor performance is by providing incentives to students for good work (e.g., stickers, stamps, points toward extra recess). If motivation is an issue, teachers should see a difference in student work when incentives are provided. If students are making errors owing to lack of knowledge rather than lack of motivation, they will not be able to improve their work in mathematics regardless of the nature of the incentive.

Formative evaluation techniques, such as Curriculum-Based Measurement (CBM), help teachers know whether students are progressing at an acceptable rate toward mastery of their annual learning goals. For more detail on formative evaluation see Deno, Espin, and Fuchs; and Shinn, Shinn, Hamilton and Clarke, both in this volume. Many mathematics programs do not contain provisions or materials for this type of formative evaluation. This situation is unfortunate since research suggests that teachers who monitor growth over time (a) use more specific, acceptable achievement goals; (b) are more realistic and less optimistic about goal attainment; (c) cite more objective and frequent data sources; (d) modify student programs more frequently; and (e) use more effective instructional variables (L. S. Fuchs & D. Fuchs, 1991; Shinn, 1989). In other words, teachers who use CBM to monitor growth are more aware of students' actual performance, and with this awareness, they are able to modify their instruction more rapidly and effectively (e.g., L. S. Fuchs, D. Fuchs, Hamlett, Phillips, & Bentz, 1994). School psychologists might explore the possibility of helping to develop probes from the curriculum for teachers. School psychologists could also do a great service by helping to establish school or district norms for purposes of setting annual mathematics goals.

Summary

School psychologists can help teachers navigate through their mathematics programs by helping them establish reasonable goals, evaluate the quality of the instructional strategies recommended in the programs, and assist teachers in choosing and implementing assessment procedures that will help them make important instructional decisions. To summarize our recommendations and assist school psychologists in helping teachers analyze and modify their mathematics programs, we have included Curriculum Analysis Guidelines in Figure 2, on page 584. These guidelines are derived from the three curricular areas of instructional goals, instructional strategies and informal assessment discussed in previous sections of this chapter. School psychologists can use these guidelines with classroom teachers as a framework for helping teachers examine their current instructional practice.

CONCLUSION

In this chapter we have suggested that the role of the school psychologist in helping schools, districts, or individual classroom teachers improve the mathematics performance of their low-achieving students is twofold. First, school psychologists can serve as an important resource providing information to administrators and teachers regarding research-based schoolwide mathematics interventions. Many school psychologists serve on committees charged with generating solutions to problems of poor school performance. When they are consulted about possible options for improving mathematics performance, school psychologists need to be knowledgeable about those interventions that have demonstrated success in improving the mathematics performance of a diverse group of students.

The second role we have proposed for the school psychologist is that of consultant to individual classroom teachers. Given the lack of preparation of most teachers in the area of mathematics and the questionable quality of mathematics curricula, school psychologists can serve as critical collaborators with teachers searching for ways to improve their mathematics instruction. The mathematics achievement of low-achieving students can be increased with the coordinated efforts of teachers, administrators, and school psychologists. Most effective interventions include a well-designed or modi-

FIGURE 2

Curriculum Analysis Guidelines

Curriculum Analysis Guidelines

I Instructional Goals

A. Are student learning goals focused on skills that are frequently used?
B. Are student learning goals focused on "big ideas?"

II Instructional Strategies

A. Are the recommended strategies of intermediate generality – not too narrow or too broad?
B. Are the recommended strategies generalizable or do they involve rote instruction?
C. Has the teacher identified the prerequisite knowledge required by the instructional strategy?
D. Has the teacher scaffolded the strategy instruction by providing the greatest amount of support initially and gradually reducing that support?
E. Has the teacher provided adequate opportunities for review of newly introduced concepts or skills?

III Informal Assessment

A. Does the teacher include both criterion-referenced and progress monitoring assessment options?
B. Does the teacher conduct an analysis of student errors to examine the cause of the errors? Has the teacher determined whether the errors are caused by lack of fact mastery, lack of strategy knowledge, lack of prerequisite knowledge, or lack of motivation?

fied mathematics program that includes a systematic means of assessing student performance. School psychologists can have a significant role in identifying effective programs or helping teachers modify programs so that more students will experience success.

REFERENCES

Adams, G. L., & Engelmann, S. (1996). *Research on Direct Instruction: Twenty-five years beyond Distar.* Seattle, WA: Educational Achievement Systems.

Becker, W. C., & Gersten, R. (1982). A follow-up of Follow Through: The later effects of the Direct Instruction Model on children in fifth and sixth grades. *American Education Research Journal, 19*(1), 75-92.

Bracey, G. (2000). Trying to understand teaching math for understanding. *Phi Delta Kappan, 81*(6), 473-474.

Cardelle-Elawar, M. (1995). Effects of metacognitive instruction on low achievers in mathematics problems. *Teaching and Teacher Education, 11,* 81-95.

Carnine, D., Dixon, R., & Silbert, J. (1998) Effective strategies for teaching mathematics. In D. Carnine & E. Kame'enui (Eds.), *Effective teaching strategies that accommodate diverse learners.* Upper Saddle River, NJ: Merrill.

Carnine, D., Grossen, B., & Silbert, J. (1995). Direct Instruction to accelerate cognitive growth. In J. Block, S. Everson, & T. Guskey (Eds.), *Choosing research-based school improvement programs* (pp. 129-152). New York: Scholastic.

Carnine, D. W., & Stein, M. (1981). Organizational strategies and practice procedures for teaching basic facts. *Journal for Research in Mathematics Education, 12*(1), 65-69.

Charles, R. I. (1980). Exemplification and characterization moves in the classroom teaching of geometry concepts. *Journal for Research in Mathematics Education, 11*(1), 10-21.

Colvin, R. (1999). Math wars: Tradition vs. real-world applications. *School Administrator, 56*(12), 26-31.

Davis, R. B. (1990). Discovery learning and constructivism. In R. B. Davis, C. A. Maher, & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Dempster, F. N. (1991). Synthesis of research on reviews and tests. *Educational Leadership, 48*(7), 71-76.

Engelmann, S. & Carnine, D. (1991). *Theory of instruction: Principles and applications.* Eugene, OR: ADI Press.

Fantuzzo, J. W., Polite, K., & Grayson, N. (1990). An evaluation of reciprocal peer tutoring across elementary school settings. *Journal of School Psychology, 28,* 309-323.

Fuchs, L. S., & Fuchs, D. (1991). Curriculum-Based Measurements: Current applications and future directions. *Preventing School Failure, 35*(3), 6-11.

Fuchs, L. S., Fuchs, D., Hamlett, C. L., Phillips, N. B., & Bentz, J. (1994). Classwide curriculum-based measurement: Helping general educators meet the challenge of student diversity. *Exceptional Children, 60*(6), 518-37.

Gersten, R., & Carnine, D. (1984). Direct instruction mathematics: A longitudinal evaluation of low SES elementary students. *Elementary School Journal, 84*(4), 395-407.

Gersten, R., Woodward, J., and Darch, C. (1986). Direct Instruction: A research-based approach to curriculum design and teaching. *Exceptional Children, 53*(1), 17-31.

Gleason, M., Carnine, D., & Boriero, D. (1990). Improving CAI effectiveness with attention to instructional design in teaching story problems to mildly handicapped students. *Journal of Special Education Technology, 10*(3), 129-136.

Good, T. L., & Grouws, D. A. (1979). The Missouri Mathematics Effectiveness Project: An experimental study in fourth-grade classrooms. *Journal of Educational Psychology, 71,* 355-362.

Good, T. L., Grouws, D. A., & Ebmeier, H. (1983). *Active mathematics teaching.* New York: Longman.

Good, T. L., Reys, B. J., Grouws, D. A., & Mulryan, C. M. (1989/1990). Using work-groups in mathematics instruction. *Educational Leadership,* 56-62.

Greenwood, C. R., Delquadri, J. C., & Hall, R. V. (1989). Longitudinal effects of classwide peer tutoring. *Journal of Educational Psychology, 81*(3), 371-383.

Greenwood, C. R., Terry, B., Utley, C. A., Montagna, D., & Walker, D. (1993). Achievement, placement, and services: Middle school benefits of classwide peer tutoring used at the elementary school. *School Psychology Review, 22*(3), 497-516.

Kame'enui, E., & Carnine, D. (1998). *Effective teaching strategies that accommodate diverse learners.* Upper Saddle River, NJ: Merrill.

Kohler, F. W., Ezell, H., Hoel, K., & Strain, P. S. (1994). Supplemental peer practice in a first grade math class: Effects on teacher behavior and five low achievers' responding and acquisition of content. *The Elementary School Journal, 94*(4), 389-403.

Leinhardt, G. (1987). Development of an expert explanation: An analysis of a sequence of subtraction lessons. *Cognition and Instruction, 4*(4), 225-282.

Loveless, T. & Diperna, P. (2000). How well are American students learning: Focus on math achievement. *The Brown Center Report on American Education, 1*(1), 1-36.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States.* Mahwah, NJ: Erlbaum.

Moore, L. J., & Carnine, D. W. (1989). A comparison of two approaches to teaching ratio and proportions to remedial and learning disabled students: Active teaching with either basal or empirically validated curriculum design material. *Remedial and Special Education, 10*(4), 28-37.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Available Internet: *http://standards.nctm.org/index.htm.*

Nattiv, A. (1994). Helping behaviors and math achievement gain of students using cooperative learning. *The Elementary School Journal, 94*(3), 285-297.

Nickerson, R. S. (1985). Understanding. *American Journal of Education, 93,* 201-239.

Phillips, N. B., Fuchs, L. S., & Fuchs, D. (1994). Effects of classwide curriculum-based measurement and peer tutoring: A collaborative researcher-practitioner interview study. *Journal of Learning Disabilities, 27*(7), 420-434.

Prawat, R. S. (1989). Promoting access to knowledge, strategy, and disposition in students: A research synthesis. *Review of Educational Research, 59*(1), 1-41.

Reese, C. M. (1996). *NAEP's 1996 Mathematics Report Card for the Nation and the States. Executive summary.* National Center for Education Statistics. Available Internet: *http://nces.ed.gov/nationsreportcard/96report/97488.pdf.*

Resnick, L. B., Cauzinille-Marmeche, E., & Mathieu, J. (1987). Understanding algebra. In J. A. Sloboda & D. Rogers (Eds.), *Cognitive processes in mathematics* (pp. 169-203). Oxford: Clarendon Press.

Resnick, L. B., & Omanson, S. F. (1987). Learning to understand arithmetic. In R. Glaser (Ed.), *Advances in instructional psychology* (pp. 41-95). Hillsdale, NJ: Erlbaum.

Riley, R. (1999). Scholars object to education department endorsement of 10 math programs for children. *The Chronicle of Higher Education, 46*(15), 37-38.

Shinn, M. R. (1989). *Curriculum-Based Measurement: Assessing special children.* New York: Guilford Press.

Slavin, R. E. (1983). *Cooperative learning.* New York: Longman.

Slavin, R., & Karweit, N. (1984). Mastery learning and student teams: A factorial experiment in urban general mathematics classes. *American Education Research Journal, 21,* 725-736.

Slavin, R., Leavey, M., & Madden, N. (1984). Combining cooperative learning and individualized instruction: Effects on student mathematics achievement, attitudes, and behaviors. *Elementary School Journal*, *84*, 409-422.

Stebbins, L. B., St. Pierre, R. G., Proper, E. C., Anderson, R. B., & Cerva, T. R. (1977). Education as experimentation: A planned variation model (Vols. IVA, IVC). An Evaluation of Follow Through. Cambridge, MA: ABT Associates.

Stein, M., Silbert, J., & Carnine, C. (1997). *Designing effective mathematics instruction: A direct instruction approach* (3rd ed.). Upper Saddle River, NJ: Merrill.

Stevens, R. J., & Slavin, R. E. (1995). The cooperative elementary school: Effects on students' achievement, attitudes, and social relations. *American Education Research Journal, 32*(2), 321-351.

Valverde, G., & Schmidt, W. (1998). Refocusing U.S. math and science education. *Issues in Science and Technology, 14,* 60-66.

Van Patten, J., Chao, C., & Reigeluth, C. M. (1986). A review of strategies for v14 n2 sequencing and synthesizing instruction. *Review of Educational Research, 56*(4), 437-471.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes* (M. Cole, V. John-Steiner, S. Scribner, & E. Souberman, Eds. & Trans.). Cambridge, MA: Harvard University Press.

Woodward, J., Baxter, J., & Robinson, R. (1999). Rules and reasons: Decimal instruction for academically low achieving students. *Learning Disabilities Research & Practice, 14,* 15-24.

Wu, H. (1999). Pre-service professional development of mathematics teachers. Unpublished manuscript. Available Internet: *www.math.berkeley.edu/~wu.*