Chapter 12

MATHEMATICS AND RTI

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How Other Areas of Learning Inform Math RTI

1. How does RTI for mathematics differ from RTI for reading?

In concept, RTI for mathematics does not differ from RTI for reading. The premise is the same: to provide *all* students with research-supported effective instruction in order to prevent: 1) student underachievement and school failure; and 2) over-identification of students for special education services. In terms of overall process, there is little difference. The basic structural components (e.g., screening, tiered instruction, use of continuous progress monitoring and data-based decision-making) of the framework are not different.

Differences lie with the specific instructional and assessment practices implemented, and obviously in the content that is emphasized. Additionally, school personnel require different areas of expertise for implementing effective mathematics RTI. To implement mathematics RTI in the most effective manner, school-level expertise in the following areas is critical:

- Teachers who are knowledgeable of the pertinent mathematics curriculum (e.g., elementary or secondary level) and how it connects to the rest of the K-14 mathematics curriculum.
- Teachers who have expertise in the application of mathematics in ways that are developmentally appropriate and relevant to students' lives.
- Teachers who have expertise in instructional practices that make abstract concepts meaningful for students.
- Specialists who can integrate mathematics instructional practices in differentiated ways based on the needs of individual students.
- Faculty and staff who are collaborative and flexible in meeting the needs of all students, from high achieving students to struggling students.
- Leaders who value these personnel characteristics, who can allocate the most critical resources needed to support mathematics RTI, and who value innovation.

Leaders of schools who value innovation are especially important right now. The knowledge base in terms of effective practices in mathematics RTI is clearly limited. Effective school leaders must have the vision and ability to empower school personnel to be consumers of the latest research and to be comfortable with changing traditional structures that do not work.

2. Aside from the basic components of RTI (mentioned above), are there concepts or practices that have been shown to be effective with reading/ literacy RTI that can inform mathematics RTI?

There are several similarities between what works in terms of reading/literacy practices and what can work for mathematics in RTI. Some basic elements should be implemented, such as using explicit instruction, scaffolding, teaching of strategies, providing multiple practice opportunities, implementing progress monitoring, and using data to inform instructional decisions. Several other instructional processes from the reading/literacy area can be applied to mathematics as well. One such process is the integration of meaning/context within mathematics (e.g., using narrative text and video that illustrates mathematical situations) and to target particular components of mathematics that are essential building blocks for later success in mathematics — similar to the importance of building phonemic and phonological awareness in reading (Gersten and Chard, 1999). The areas of number sense and algebraic thinking are two possibilities for special emphasis.

Placing special emphasis on key foundational mathematics concepts or big ideas that situate students to make meaning of mathematics is highly recommended. Rather than trying to cover every benchmark or sub-skill associated with the K-12 mathematics curriculum, focusing on a smaller set of concepts/processes/skills that are foundational to mathematical literacy seems to be a more logical approach. Such a focus can reduce the number of individual benchmarks incorporated within a mathematics curriculum, allowing more time for students to fully acquire understanding, develop proficiency, and obtain mastery of what is most important related to the development of mathematical literacy. In turn, such an approach better situates students to generalize and even adapt the mathematical big ideas for other uses and to develop new knowledge. If your particular state standards do not include mathematical big ideas for each grade level, using the focal points emphasized by the National Council of Teachers of Mathematics can be an efficient way to conceptualize the essential big ideas relative to your curriculum.

Another process from the reading/literacy area that can be effectively applied to mathematics is matching learning contexts to the level of understanding students currently possess. In reading, students read texts that are written and formatted to match their current level of understanding of text and how it relates to the spoken word (e.g., number of words, length of words, repeated words/phrases/ rhymes, use of pictures, etc.). In mathematics, a similar process can be emphasized using a concrete-representational-abstract sequence of assessment and instruction for targeted mathematical big ideas.

Like letters and punctuation marks, numbers and mathematical symbols have no meaning in and of themselves. Students must have something meaningful to attach to these abstract symbols. Working with students to develop meaning, and ultimately understanding, of what mathematical symbols, expressions, statements and processes represent through experiences with materials (concrete level), then through drawings (representational level), and finally with only numbers and symbols without use of materials or drawings (abstract level) provides a supported sequence of learning contexts to develop mathematical understanding similar to sequenced texts in reading. At the same time, continuous progress monitoring at the concrete, representational and abstract levels of understanding is critical, particularly for students learning in Tier 2 and Tier 3 contexts.

Finally, it's important for those entrusted with K-12 mathematics outcomes to consider how the disciplines of special education and reading/literacy education have "joined together" to improve the effectiveness of reading instruction overall for struggling learners. Example: the concept of a "balanced" approach to reading, where essential aspects of explicit reading instruction (special education) and aspects of whole language (reading/literacy education) are blended and scaffolded based on the learner's needs. Similarly, encourage collaboration among special educators and mathematics educators so that similar breakthroughs in mathematics can also occur.

3. Are there core components essential to mathematics skill development that should be emphasized within mathematics RTI, similar to the five core components of reading?

There seems to be some correlation between what has been learned in reading and what it may mean for mathematics. An area that seems to be most critical for K-12 mathematics success is the area of number/number sense. For example, lack of number sense seems to be a consistent issue for students who fail algebra at the secondary level. Placing more emphasis on this important area where students have multiple opportunities to truly develop meaning for numbers, how numbers relate, and how numbers can be manipulated to represent different ideas is highly suggested. Research suggests that fluency with basic computations is a predictor of later mathematical success. It is logical to conclude that students who are fluent with computations have developed a reasonable level of number sense and can apply it in different ways to compute efficiently.

While fluency with computation is important for later mathematical success, it is only an indicator of potential success. Primary emphasis should be placed on ensuring students acquire understandings of the characteristics of numbers, how numbers relate, and how they can be manipulated to communicate ideas and to solve problems. Providing a systematic and meaningful process for developing such understandings through concrete-representational-abstract instruction, combined with use of authentic contexts that naturally have certain number sense applications imbedded within them, and using language to help students connect meaning to mathematics, are excellent instructional processes for developing number sense and fluency with number operations (Witzel, Ferguson, and Brown, 2007).

An emphasis on algebra throughout the K-8 years is also recommended (The Final Report of the National Mathematics Advisory Panel, 2008), including an emphasis on fluency with whole numbers, fluency with fractions, and particular aspects of geometry and measurement that are required for algebra and algebraic thinking (e.g., properties of two- and three-dimensional shapes and line slope/linear functions).

Students, particularly those who are struggling to learn mathematics, should be systematically exposed to multiple processes of doing mathematics. The National Council of Teachers of Mathematics suggests five processes for doing mathematics: reasoning/proof, representation, communication, connections and problem-solving. Too often, students who struggle get exposed to only procedural mathematics (i.e., computation) when other processes for doing mathematics may actually assist with strengthening computation abilities by strengthening students' conceptual understandings.

Doing mathematics in different ways (e.g., by communicating what one understands about a particular mathematical concept or representation, by representing a mathematical concept or abstract representation using drawings, tables or with materials, and by showing how two mathematical concepts connect through use of graphic organizers) can help students to use and develop metacognitive thinking skills, an important component of problem-solving and something struggling learners often do not apply. Metacognitive awareness is a skill for which these students need explicit instruction.

Assessment & Progress Monitoring

Screening

4. What considerations should be made when selecting a screening instrument for mathematics?

Fuchs (2006) suggests four considerations when selecting a mathematics screening instrument. First, the instrument should be feasible to implement — in the amount of time it takes to implement it, in the level of preparation/training required for personnel to administer it, and in terms of cost.

Second, the results from the instrument should be a strong predictor of how students will perform on relevant high stakes testing. That is, if a student scores low on the screening instrument, he would score low on the high stakes test.

Third, the mathematics content that is measured should be developmentally appropriate for students. The content included in a screening instrument should represent what students have already had the opportunity to master, given their grade or age. Instruments that include an overreliance of items that are too basic or too advanced will provide inaccurate appraisals of students' abilities.

Fourth, the screening instrument should have an accurate cut-score for determining if students will require extra support. A selected instrument should satisfy these four considerations at minimum, whether or not it is a criterion-referenced instrument (where an absolute value for a particular skill — e.g., certain number of items correct — is correlated with successful future performance on that skill) or a norm-referenced instrument used (where performance above a certain score or percentile rank compared to a normative sample is correlated with successful future performance).

In addition to these considerations, remember that whichever assessments are in place (screening, progress monitoring, high stakes tests), they will tend to drive the emphasis on instruction. For example, if procedural types of mathematics (e.g., computation) are emphasized over other processes of doing mathematics (e.g., problem-solving, reasoning/proof, connections, communication, representation) then it is very likely that procedural mathematics will be the focus of teacher instruction and vice versa. Therefore, it is important to evaluate the extent to which a screening instrument and any other assessment instrument relates to the processes for doing mathematics desired by a school or district in addition to the content (e.g., numbers and operations, algebra, geometry, data analysis and probability, measurement.). Some researchers have suggested that an area of focus for early screening in mathematics (K-3) is numerical proficiency (Gersten, Clare, and Jordan, 2007).

5. Should scores from the prior year's state assessment in mathematics be used as a screening instrument for the subsequent year?

While student performance on the previous year's state assessment can be relevant information, it is not recommended for use as the primary data for screening purposes. There are several reasons for this. The amount of lapsed time between the prior year's state assessment and the beginning of the current school year is too great to make reasonable inferences. Some students will further develop their mathematical knowledge during that time through extended day programs, individual tutoring experiences, or help from family members. Students who receive year-round schooling would naturally have additional mathematics learning experiences. Some students may experience a decrease in their mathematical abilities due to a lack of mathematical experiences. Likewise, students may have been able to demonstrate some level of competency for certain mathematical knowledge and skill areas when the state assessment was administered but did not have sufficient practice opportunities during the remainder of the school year to become proficient.

Without additional practice, students will have difficulty replicating their performance after the lapse of time between last year's state assessment and the beginning of the next school year. Another concern is the extent to which a state mathematics assessment administered at one grade level correlates with performance at the next grade level's administration. Such a correlation would need to be evident in order to consider using it even as one source of data in a RTI screening process for mathematics.

Continuous Progress Monitoring

6. Is there a need for mathematics progress monitoring probes that measure ways of doing mathematics other than computation fluency?

Ideally it would be reasonable to incorporate multiple mathematical processes (ways of doing mathematics) for progress monitoring. With that said, there are several factors that have made this difficult to do. Only a small amount of research and development has been done in this area for mathematics. To date, the vast majority of available probes are computation driven and emphasize fluency at the abstract level of understanding (rate and accuracy of computations). Therefore, this is what many schools and districts are using presently.

An exception to this is the work in curriculum-based measurement led by Fuchs and Fuchs and their colleagues at Vanderbilt University and the National Research Center for Learning Disabilities (*Using Curriculum Based Measurement for Progress Monitoring in Math*, 2007). They have developed a series of K-8 mathematics probes that include computation and application-type items. A summary of their work, with examples of probes, is provided in *Using Curriculum Based Measurement for Progress Monitoring in Math*. Download a copy at the National Center on Student Progress Monitoring Web site, *www.studentprogress.org*. For schools currently looking for mathematics CBM probes that are already developed and that include more than computation type skills, investigate the mathematics progress monitoring probes developed at Vanderbilt and the National Research Center for Learning Disabilities to determine the extent to which they address the types of mathematics knowledge and skills of interest.

Another factor is that of practicality. It takes longer for students to use processes such as problem-solving, reasoning/proof, communication, representation and connections. To be effective, continuous progress monitoring must be done frequently, take only small amounts of time for administration and scoring, and produce data that is comparable across students, grade levels, schools, etc. Fluency has been the primary measure in progress monitoring historically, therefore this is the measure that has received the most attention in terms of research and validation.

My colleagues and I are in the beginning stages of developing mathematics probes that address multiple mathematics processes in algebra/algebraic thinking. For processes such as representation and communication, the measure is accuracy with one or a few items, rather than fluency with many items. A rubric is used to evaluate the degree to which a student is using a process accurately for a given mathematics concept or skill. Ratings (e.g., 1-4) per probe are plotted on a chart, just as the number correct or the percentage correct is plotted for fluency. The criterion for mastery is consecutive number of attempts where a student receives the highest rating (e.g., a rating of four out of four for three to five sessions).

Additional work is needed in order to determine the extent to which such an approach is viable for use in schoolwide and districtwide curriculum-based measurement systems. Certainly, the nature of probes used for continuous progress monitoring in mathematics and RTI is an important area for further development and evaluation on the part of researchers.

7. When making data-based instructional decisions for mathematics RTI, are there particular factors that educators should consider?

There are several different RTI data-based decision-making approaches that are advocated in the literature and implemented in schools. The Standard Protocol (RTI-SP) approach emphasizes the degree to which empirically validated instructional practices are implemented using standardized and validated observation fidelity protocols.

If a student is receiving empirically validated instruction appropriately as determined by a score or rating using an observation protocol and is demonstrating a lack of progress based on CBM, then it is assumed that a change in instruction is needed. In this approach, little analysis of the deficit skill and what the student does or doesn't know about it occurs. The Problem-solving (RTI-PS) approach differs in that the central target is on the concepts/skills students are having difficulty with and shaping instruction to address students' learning needs (Crist, Burns, and Ysseldyke, 2005).

Both approaches assume the use of evidence-based or research-supported practices and are applied systematically. Another approach, the Progressive Intervention (RTI-PI) approach, combines RTI-SP and RTI-PS in a scaffolded manner (O'Shaughnessy, 2003). At Tier 1, an RTI-SP approach is used. Thereafter, as students demonstrate more difficulty at Tier 2 and Tier 3, an RTI-PS approach is used.

Experts differ on which approach is the most effective. Some believe the RTI-SP approach is superior, while others prefer the RTI-PS or RTI-PI approach. Several considerations should be made when determining which approach to use for mathematics and RTI. One consideration is the nature of the mathematics instruction that is being implemented at each tier. If a validated mathematics program is being implemented and it includes an appropriately validated fidelity protocol, then an RTI-SP approach might be considered. However, if such a program is not being implemented, then an RTI-PS might be a better choice. If a validated program is being used at only the Tier 1 level, then the RTI-PI approach may be most effective.

A second consideration for mathematics is the availability of validated instructional programs that appropriately address the mathematics curriculum of a particular school or district. Currently, there are very few available and those that are available do not address the content or learning needs of all learners K-12. Therefore, most schools will likely need to consider where in the curriculum such programs can be used effectively and where in the curriculum research-supported effective mathematics practices will be implemented that are done so outside of the context of a validated published program. When these decisions are made, then the best approach or approaches to instructional decision-making can be made.

A third consideration is the level of expertise a school might have related to the mathematics curriculum and in understanding the needs of diverse students. The RTI-SP approach assumes less need for such expertise since decisions are made based on whether or not the program is being implemented accurately and whether or not students are meeting set performance criteria. The RTI-PS and RTI-PI approaches require personnel with appropriate expertise in mathematics education, personnel with expertise in instruction that meets the needs of exceptional learners including struggling learners and learners who are gifted/talented in mathematics, and personnel with expertise in the learning needs of students who have limited English proficiency. Therefore, at the building level in particular, school leaders should evaluate their personnel in order to determine the level of expertise available related to mathematics education and the needs of their student population. Based on this evaluation, school leaders will be in a better position to determine the approach that will be most effective for mathematics RTI.

Assessment & Continuous Progress Monitoring Resources

8. What other resources for mathematics probes can be used for continuous progress monitoring?

Here are a few:

- *National Center on Progress Monitoring* (*www.studentprogress.org*) – This project sponsored by the US Office of Special Education Programs (IDEAs that Work) maintains an Internet site that provides a wealth of information on effective practices related to progress monitoring. Their "Web Sources" link provides many easily accessible resources.
- *Aimsweb* (*www.aimsweb.com*) – This Pearson Publications Internet resource is a for-fee, Web-based progress monitoring system for RTI.
- *Curriculum-based Measurement Warehouse (Intervention Central)* – provides information on and links to multiple Internet sites related to curriculum-based measurement including mathematics probes. *www.interventioncentral.org/htmdocs/interventions/cbmwarehouse.php*

Instruction

Tiered Instruction

9. How is tiered mathematics instruction provided?

Three key principles of tiered instruction, as it relates to accessing the general education curriculum, are: 1) all students receive effective instruction based on sound research (i.e., instruction that results in expected outcomes for students); 2) increasing levels of support and explicitness are provided as students demonstrate difficulties; and 3) opportunities for students to move back and forth among tiered instructional levels are provided based on their current needs.

Most models in RTI have three instructional tiers, however some models incorporate more. For purposes of this discussion, a three-tier model is assumed. The terms "intensity" (the amount of instructional support provided, including reduced teacher-to-student ratios in the form of small group and one-to-one instruction) and "duration" (amount of time instruction occurs) are often used as descriptors for what differentiates instruction at each tier. Instruction at all three tiers should incorporate research-supported instructional practices. Generally, Tier 2 instruction should be more intensive and occur for a longer duration compared to instruction at Tier 1. Likewise, instruction at Tier 3 should be more intensive and occur for a longer duration than instruction at Tier 2.

The structure in which mathematics instruction is provided at each tier can vary. Some RTI mathematics models implement tiered instruction in separate instructional contexts. Tier 1 instruction occurs at the whole classroom level. Tier 2 instructional models most commonly occur in small group settings (e.g., 2-6 students) separate from the whole class setting. Similarly, Tier 3 instruction models incorporate one-to-one instruction separate from the whole class setting.

In some cases, Tier 2 and Tier 3 instructional interventions, which incorporate small group or one-to-one instruction, occur within the whole classroom context. Most likely this would happen in classrooms where two or more teachers are working collaboratively to deliver multitiered instruction.

Based on the model a particular school adopts, students receiving Tier 2 or Tier 3 instruction may or may not also receive mathematics instruction in Tier 1. For example, a student might require more intensive instruction for a set of mathematical concepts that represent small gaps in his mathematical knowledge base. Small group or one-to-one instruction for those concepts might be provided during 20- to 30-minute intervention sessions that occur during another part of the students' day, different from when the student receives Tier 1 mathematics instruction (see Bryant and Bryant, 2007).

A school might schedule a regular block of time during the day where all students engage in both remedial and extension types of learning activities based on each student's relative needs. Such a process can be structured at the class, team, grade or even school level depending on what works best given student numbers, academic schedules, and availability of support staff and volunteers. Other students might possess more in-depth mathematical knowledge gaps and therefore may not benefit from Tier 1 instruction. These students might benefit from receiving all of their mathematics instruction in small group or one-to-one instructional contexts only.

10. How does the expertise of school personnel factor into providing tiered instruction?

The nature of the expertise that personnel within a school or district possess might be the best place to start when determining how tiered mathematics instruction is structured. Utilizing teachers, support staff and specialists in ways that best fit their levels of expertise and experience is a logical place to begin. Personnel with differing levels of preparation, experience and expertise might be best suited to lead instruction at different tiers.

The levels of expertise personnel have in mathematics, in mathematics pedagogy, in teaching struggling learners, etc., are appropriate indicators for making such decisions. For example, educators with more knowledge of the total K-12 mathematics curriculum *and* who have expertise in working with struggling learners might be more effectively utilized at the Tier 2 and Tier 3 levels, whether intervention is provided within the whole classroom where Tier 1 instruction occurs (i.e., co-teach) or in separate contexts. It is more likely that teachers with this level of expertise will be able to address a wider range of mathematics knowledge and skill gaps with which students are struggling and will be more likely able to individualize instruction based on the needs of each student.

11. Is scaffolding within tiers possible?

Scaffolding support within tiers is not only possible, it's another structure schools can implement for mathematics RTI so that there is a level of differentiation within each tier. For example, in Tier 1 some classrooms would have one teacher who is experienced with implementing effective mathematics instruction practices and differentiating their use to meet all of her students' needs. This might represent Tier 1A instruction. Other classrooms would be co-teach situations, where two teachers work collaboratively to implement effective mathematics instruction that is differentiated with a small level of additional support. This might represent Tier 1B instruction.

While Tier 1 would be occurring in each type of context, the co-teach classroom would be made available for those students who need slightly more support than other students in Tier 1. Tier 2 and Tier 3 instruction would emphasize a set of additional explicit instructional practices implemented in more intensive ways. At Tier 2A, for example, small group instruction might be implemented by a math coach or teacher experienced in teaching struggling learners using the same instructional practices as Tier 1, but with more intensity and duration. At Tier 2B, small group instruction is provided again by a math coach or teacher experienced with teaching struggling learners using the same instructional intensity and duration as Tier 2A but incorporating additional explicit mathematics instructional practices. At Tier 3A and Tier 3B a similar process of scaffolding instructional support would be provided, except that instruction would occur at a one-to-one level. At this level, teachers who have expertise in working with students who have more significant learning difficulties (i.e., special educators) might work collaboratively with the math coach or intervention specialist to assist them in problem-solving, in planning, and with implementing Tier 3 instruction.

By scaffolding structures within each tier, schools can more precisely and more systematically provide increasing levels of mathematics instructional support for students and possess more flexibility in effectively utilizing personnel, space and resources. Such a structure also provides even more flexibility for adjusting the instructional context for students based on performance data because there would be smaller increments of instructional change compared to the traditional three-tier structure.

12. Regardless of the tiered structure used, do the structures share any common elements?

Yes. The tiered structure a school chooses to implement for mathematics RTI will vary based on a number of factors, including expertise of personnel, needs of students, available resources, and to some extent, the existing RTI structures that are in place for reading, behavior, and other areas of emphasis. Nonetheless, it is important that the tiered instruction structure that is implemented incorporates these three essential elements *at a minimum*: 1) all students receive effective instruction based on sound research (i.e., instruction that results in expected outcomes for students); 2) increasing levels of support and explicitness are provided as students demonstrate difficulties; and 3) opportunities for students to move back and forth among tiered instructional levels are provided based on their current needs.

13. Are there any validated Tier 2 and Tier 3 mathematics instructional interventions?

Currently, there are no Tier 2 or Tier 3 interventions that have been empirically validated for mathematics specifically. The development and empirical validation of mathematics interventions for RTI is in its infancy. However, there are a few research groups who are currently in the development and validation process for such interventions (see Bryant and Bryant, 2007). Bryant and Bryant (University of Texas) and their colleagues are currently studying the effects of an early grades (K-2) three-tier mathematics intervention with a focus on number sense and basic skill knowledge and application.

At Tier 1, a balanced approach to mathematics instruction is provided, including opportunities for students to engage in meaningful practice including peer tutoring and computer assisted instruction, a mathematically rich environment that makes mathematics transparent (emphasis on mathematics vocabulary, meaning of abstract symbols, use of manipulatives, and calculator work), explicit instruction that incorporates questioning strategies that require students to communicate their mathematical understandings, progress monitoring, an emphasis on problem-solving as well as computations, and instructional adaptations.

At Tier 2, students are provided "Booster Sessions" which occur in small groups (2-5 students). Tier 2 Booster Sessions incorporate the use of concrete-representational-abstract instruction, systematic instruction that includes framing the lesson, previewing, modeling, guided practices, independent practice checking for understanding, and error correction and feedback. As students demonstrate proficiency levels from low to high, they move through initial instruction, to skill practice, to fluency building. Sessions last approximately 20 minutes and occur four days per week. At Tier 3, similar instructional practices are implemented but are implemented with more intensity (smaller groups) and for longer duration. Fidelity of instruction is monitored through the use of fidelity monitoring forms.

Peer Assisted Learning Strategies for mathematics is a second intervention that has been demonstrated to result in positive student learning outcomes. This explicit systematic peer mediated intervention incorporates a curriculum-based measurement process that fits nicely within RTI (Fuchs, Fuchs, &

Karns, 2001). More information about PALS can be found on their Web site, *http://kc.vanderbilt.edu/pals*.

My colleagues and I are in the early stages of field-testing an intervention, *Developing Algebraic Literacy*. The focus for DAL at this point is K-3 algebra, which includes both number/number sense concepts and algebraic thinking concepts. It may have potential for Tier 2 and Tier 3 interventions. The intervention incorporates the use of contexts within which algebraic problem-solving, reasoning/proof, representation, communication and connection-making is emphasized. It uses an explicit systematic instruction process across three phases: fluency-building, measuring progress, and problem-solving the new.

The development of proficiency is the focus of the *fluency* phase. Continuous progress monitoring and data-based decision-making is the focus of the *measuring progress* phase. Application of explicit connection-making, teaching problem-solving strategies, and using language to attach meaning to new concepts and problem-solving situations is the focus of the *problem-solving the new* phase. A concrete-to-representational-to-abstract sequence of instruction is emphasized throughout. However, compared to the work of the other two initiatives described above, the validation process for the DAL is only at the beginning stages.

Effective Mathematics Instructional Practices

14. Are there particular evidence-based effective mathematics instructional practices that should be implemented for RTI?

The research base on effective mathematics instructional practices is underdeveloped compared to areas such as reading/literacy and behavioral supports. Therefore, continued research in the area of effective mathematics instruction is needed in order to better understand how students with diverse learning needs can best develop their mathematical competencies. With this said, there is a solid foundation of research to guide schools and districts as they identify and implement effective mathematics instructional practices for RTI.

There are a number of mathematics instructional practices that appear to be promising from a research perspective. They include: 1) explicit systematic instruction within authentic contexts; 2) teaching strategies for learning and doing mathematics including use of graphic organizers; 3) grounding abstract concepts within concrete experiences (concrete-representational-abstract sequence of instruction); 4) providing multiple opportunities for students to apply their mathematical understandings (both newly learned concepts and those for maintenance); and 5) continuous progress monitoring/instructional decision-making. Each of these practices have substantial enough research support as to their effectiveness, particularly for struggling learners, that any school or district should incorporate these practices at Tier 1.

Several other mathematics instructional practices that have a growing research base regarding their effectiveness can be integrated to enhance or differentiate Tier 1 instruction and to develop Tier 2 and Tier 3 interventions. They include: 1) use of explicit language experiences where students communicate their understandings about target mathematics concepts; 2) use of structured cooperative learning groups and structured classwide peer tutoring; 3) concrete-to-representational-to-abstract assessment and continuous progress monitoring; and 4) teaching the big ideas of mathematics and how they connect to one another.

The resources listed below provide additional information about research-supported effective mathematics instructional practices for further study and professional development.

Effective Mathematics Instruction Resources

15. What other resources for effective mathematics instruction are there?

Several, including those found on the Internet, syntheses of research/reviews of literature, and books:

Internet

- MathVIDS – This Web site provides comprehensive information and video models of each of these practices as well as others. Additionally, the research support for each practice is identified for each instructional practice. http://fcit.usf.edu/mathvids
- Center on Instruction – This Web site provides current reports of research and resources related to evidence-based mathematics instructional practices. www.centeroninstruction.org
- The Access Center – This Web site is sponsored by the United States Department of Education (IDEAs That Work, Office of Special Education Programs) and housed at the American Institutes for Research in Washington, D.C. This Web site is a clearinghouse for research-supported instructional practices for struggling learners. www.k8accesscenter.org/index.php/category/math
- The IRIS Center – This site provides online professional development for educators related to research-supported instructional practices for students with disabilities. http://iris.peabody.vanderbilt.edu/
- Special Connections – This Web site provides professional development modules for educators on research-supported practices for struggling learners, including students with disabilities in the areas of instruction, assessment, collaboration and behavior plans for the purpose of accessing the general education curriculum. www.specialconnections.ku.edu/cgi-bin/cgiwrap/specconn/index.php

Syntheses of Research/Reviews of Literature

- Butler, F.M., Miller, S.P., Lee, K., and Pierce, T. (2001). Teaching mathematics to students with mild-to-moderate mental retardation: A review of the literature. *Mental Retardation, 39*(1), 20-31.
- Gersten, Baker, & Chard (2006). Effective Instructional Practices for Students with Difficulties in Mathematics: Findings from a Research Synthesis. Center on Instruction, www.centeroninstruction.org
- Kroesbergen, E.H., & van Luit, J.E.H. (2003). Mathematics interventions for children with special educational needs. Remedial and Special Education, 24, 97–114.
- Maccini, P., & Hughes, C.A. (1997). Mathematics interventions for adolescents with learning disabilities. *Learning Disabilities Research & Practice,* 12, 168-176.
- Maccini, P., & Gagnon, J.C. (2000). Best practices for teaching mathematics to secondary students with special needs. Focus on Exceptional Children, 32, 1–22.
- Mastropieri, M.A., Scruggs, T.E., & Shiah, S. (1991). Mathematics instruction for learning disabled students: A review of research. *Learning Disabilities Research & Practice,* 6, 89-98.

- Miller, S.P., Butler, F.M., & Lee, K. (1998). Validated practices for teaching mathematics to students with learning disabilities: A review of the literature. Focus on Exceptional Children, 31, 1–24.
- Montague, M. (2006). Self-regulation strategies for better math performance in middle school. In M. Montague & A. K. Jitendra (Eds.), Teaching mathematics to middle school students with learning difficulties (pp. 89-107). New York: Guilford Press.
- Swanson, H.L. (1999). Instructional components that predict treatment outcomes for students with learning disabilities: Support for a combined strategy and direct instruction model. *Learning Disabilities Research & Practice*, 14(3), 129–140.
- Swanson, H.L., & Hoskyn, M. (2001). A meta-analysis of intervention research for adolescent students with learning disabilities. *Learning Disabilities Research & Practice*, 16, 109-119.
- Vaughn, S., Gersten, R., & Chard, D.J. (2000). The underlying message in LD intervention research: Findings from research syntheses. Exceptional Children, 67, 98–114.

Books

- Allsopp, D.H., Kyger, M.M., & Lovin, L. (2007). *Teaching Mathematics Meaningfully: Solutions for Reaching Struggling Learners.* Baltimore: Paul H. Brookes Publishing, Co.
- Hudson, P. and Miller, S. (2007). *Designing and Implementing Mathematics to Students with Diverse Learning Needs.* Boston: Allyn & Bacon, an imprint of Pearson Education.

16. Are there evidence-based mathematics curricula and programs available that already incorporate effective mathematics instructional practices?

The *Best Evidence Encyclopedia (BEE)*, a United States Department of Education-funded project at John's Hopkins University, is a valuable source for helping schools and districts determine the extent to which certain mathematics curricula and programs have an evidence base as to their effectiveness (*www.bestevidence.org*). Each curriculum/program is rated based on a review of the quality of and the findings of empirical research studies that have been completed. Separate reviews are provided for programs at the elementary and secondary levels. Programs are identified as having a strong evidence base, a moderate evidence base, insufficient evidence base, or no evidence base (no qualifying studies have been conducted to evaluate). The What Works Clearinghouse (*http://ies.ed.gov/ncee/wwc*), sponsored by the Institutes for Educational Science (US Department of Education), provides a similar service for elementary and middle school mathematics.

It is important to remember that while a particular program may have been demonstrated to be effective in a small set of studies, assuming that similar levels of effectiveness will occur within one's own school context should be met with caution. The reality is that many programs that have been reviewed are not comprehensive in nature because they focus on a subset of the K-12 mathematics curriculum. Moreover, the nature of the learning outcomes measured may not represent your interests or your students' needs. For example, one program may emphasize computation with little emphasis on problem-solving while another emphasizes problem-solving with little emphasis on computation. Student differences (e.g., level and type of diversity, existing mathematics learning problems, nature of knowledge gaps) and teacher differences (e.g., level of teacher preparation, expertise/understanding of the mathematics teachers are teaching, school and classroom climates/contexts) between participants in studies reviewed and one's own situation also make direct comparisons difficult.

17. How can existing mathematics curricula be modified to integrate effective mathematics instructional practices that address the needs of struggling learners?

While a review process such as that completed by the BEE or the What Works Clearinghouse can be helpful by identifying *whole programs* that have some evidence base for their effectiveness, it does not inform educators regarding the extent to which a particular curriculum or program incorporates some or all of available research-supported effective mathematics instructional practices, like those identified previously (i.e., question #15). Whether a program is effective for a particular school or group of students is dependent on *how* the program is implemented, the extent to which it appropriately matches the content and learning needs of students, and the extent to which instructional adaptations and modifications are made as needed.

Schools and districts should base their evaluation of mathematics curricula/programs for adoption on the current evidence base for a particular curriculum/program (e.g., BEE) and the extent to which research-supported mathematics instruction practices are integrated within the curriculum/program (see previous question). Doing this will help educators make more informed decisions about mathematics curricula selected for use and it will provide them a way to augment an adopted program by integrating effective mathematics instructional practices that are not included or that are not emphasized in a comprehensive way.

The guide titled *Mathematics Curriculum Evaluation Guide for Struggling Learners* in Appendix C - Resources is a simple checklist that shows how mathematics curricula/programs might be evaluated regarding its inclusion of research-supported instructional practices. This checklist pertains to effective mathematics instructional practices for struggling learners.

18. Can RTI mathematics be applied to high school students as well as elementary schools?

This is an important question. Unfortunately, it is also one where clear answers are not readily available. The best that can be said is that this area is a "work in progress." The implementation of Mathematics RTI generally (at the elementary or secondary level) is not nearly as advanced as it is, say, for reading/literacy or behavior. Even so, much less has been done at the high school level compared to the elementary school level. Compounding the issue at the secondary level is that mathematics becomes much more specialized in terms of content. Additionally, graduation requirements and state assessments are tied to particular mathematics courses (e.g., Algebra 1, Geometry 1, etc.). Wide-scale applications of mathematics RTI at the secondary level are just not available to provide schools much guidance. This is not to say that RTI is not happening at some high schools. However, in terms of systematically applied and evaluated models, none are available at this time.

The good news is that there is a research base from which secondary educators can base their tiered instruction. There are a number of mathematics instructional practices that appear to be promising from a research perspective, all of which can be applied to secondary settings. They include: 1) explicit systematic instruction within authentic contexts; 2) teaching strategies for learning and doing mathematics including use of graphic organizers; 3) grounding abstract concepts within concrete experiences (concrete-representational-abstract sequence of instruction); 4) providing multiple opportunities for students to apply their mathematical understandings (both newly learned concepts and those for maintenance); and 5) continuous progress monitoring/instructional decision-making. For a recent synthesis of research on effective mathematics instruction for students with mathematics difficulties, see Gersten, Baker, & Chard (2006). Effective Instructional Practices for Students with Difficulties in Mathematics: Findings from a Research Synthesis. Center on Instruction, *www.centeroninstruction.org*.

The following articles address effective mathematics instruction for struggling learners/students with disabilities at the secondary level:

- Gagnon, J.C. & Maccini, P. (2001). Preparing students with disabilities for algebra. Teaching Exceptional Children, 34(1), 8-15.

- Maccini, P., & Hughes, C.A. (1997). Mathematics interventions for adolescents with learning disabilities. *Learning Disabilities Research & Practice,* 12, 168-176.

- Maccini, P. & Gagnon, J.C. (2000). Best practices for teaching mathematics to secondary students with special needs: Implications from teacher perceptions and a review of the literature. Focus on Exceptional Children, 32(5), 1-22.

- Montague, M. (2006). Self-regulation strategies for better math performance in middle school. In M. Montague & A. K. Jitendra (Eds.), Teaching mathematics to middle school students with learning difficulties (pp. 89-107). New York: Guilford Press.

- Swanson, H.L., & Hoskyn, M. (2001). A meta-analysis of intervention research for adolescent students with learning disabilities. *Learning Disabilities Research & Practice,* 16, 109-119.

Identification of Mathematics Disabilities/Special Education Services

19. What tend to be the specific causes for poor performance in math?

Poor performance in mathematics can occur for a number of reasons. For example, it may be due to ineffective teaching practices. It could be due to factors related to the richness of a student's learning experiences before beginning school. It could be the use of a curriculum approach and pacing guide that does not provide students adequate time to acquire an understanding of early mathematics concepts and processes or ample opportunities to apply their developing knowledge in order to become proficient and to master the content.

Some students have mathematics disabilities that result in learning characteristics that serve as barriers to learning mathematics. These learning characteristics can include learned helplessness, anxiety toward learning mathematics, a passive approach to learning mathematics, attention deficits, memory problems, metacognitive deficits, and cognitive processing deficits.

For some students with learning disabilities, language deficits can affect students' abilities to make sense of the abstract because they are apt to not use language to communicate mathematically. These students tend to have difficulty with developing conceptual understanding of mathematics and experience it as a very rote and non-meaningful process.

A small portion of students with learning disabilities have significant visual-spatial-motor integration difficulties which make abstract level number and symbol manipulation extremely difficult. Oftentimes, this subset of students with learning disabilities have the most difficulty developing number sense, fluency with computations, and doing mental mathematics.

20. Is it likely that students with mathematics learning difficulties will benefit from instruction in Tier 1, Tier 2 or Tier 3?

The vast majority of students who have mathematics learning difficulties can learn mathematics effectively and will greatly benefit from the scaffolded levels of instructional support provided across instructional tiers. However, in order for these students to be successful they will require mathematics instructional practices that address their learning needs (see the earlier question regarding research-supported effective mathematics instructional practices).

The field is in the beginning stages of developing differentiated structures for delivery of these effective practices, especially at the Tier 2 and Tier 3 levels. The initial outcomes from a study that is following a group of students from first through third grade suggests that after one year of Tier 2-type of systematic intensive intervention (small group tutoring and individual computer assisted instruction) that the number of students demonstrating mathematics disabilities decreased (Fuchs, Compton, Fuchs, Paulsen, Bryant, and Hamlett, 2005). Unfortunately, the research base on the extent to which students benefit from any particular Tier 3 intervention in mathematics is too limited to draw any conclusions (see Coleman, Buysse, & Neitzel, 2006).

However, there is enough research support, both from the special education and the mathematics education literature, that provides us a solid foundation for integrating effective practices that can pay dividends for students across all tiers, including Tier 3 (e.g., explicit instruction within CRA sequence; teaching problem-solving strategies; scaffolding; authentic contexts; building connections; language experiences; multiple opportunities to apply knowledge in a variety of contexts; progress monitoring and use of data to make instructional decisions; emphasis on multiple processes for doing and expressing mathematics). Continued research should be directed toward how these effective practices can be most efficiently and effectively delivered at Tier 1, Tier 2 and Tier 3.

21. What specific diagnostic tests would you suggest to determine a true math disability such as dyscalculia, and how would the condition be treated?

There has been quite a lot of discussion about what math learning disabilities are, how they are similar and dissimilar to other learning disabilities such as learning disabilities that affect reading, and how to assess their occurrence. Traditionally, a discrepancy model has been used to determine learning disabilities in mathematics as well as other areas of learning. This typically involves looking for a significant difference between a student's IQ score and one or more scores on subtests of a mathematics achievement battery. Additionally, some have also examined significant discrepancies among achievement and cognitive processing batteries; that is, looking for any obvious inconsistencies among particular areas of mathematical abilities (calculation, application) and cognitive processing (e.g., visual, auditory, motor, processing speed, etc.). The validity of the discrepancy model has been called into question in recent years. Some researchers suggest that students with math learning disabilities have common areas of difficulty, including difficulty with numeracy, slow processing speed, and difficulty with working memory.

Recently, RTI has been suggested as a possible alternative to identifying mathematics learning disabilities. Fuchs, Compton, Fuchs, Paulsen, Bryant, & Hamlett (2005) suggest its use as a process for identifying mathematics disabilities at the end of the first grade year. They found that the best predictors for identification were: 1) low performance on end-of-the-year mathematics achievement tests that assess first grade concept application and computation; and 2) poor rate of growth across the year using curriculum-based measurement. It is important to understand that the term *math disability* does not refer to one particular type of mathematics difficulty. Mathematics learning disabilities are multifaceted and can be the result of a variety of problematic areas of learning. In addition to those areas already mentioned, language difficulties can be an issue, as can visual-spatial-motor processing deficits and sequencing.

A multi-lens perspective is likely the best approach at this point in terms of assessment for mathematics learning disabilities. It is reasonable to place special emphasis on areas such as numeracy/number sense, processing speed, working memory, and combined performance on end-of-year achievement tests and yearlong growth demonstrated via CBM. Any assessment should include both conceptual application of mathematics (i.e., reasoning, representation, communication, connections, problem-solving) and computation. For more information on mathematics disabilities including dyscalculia and assessment, visit the math link at the LD Online® Web site, *www.ldonline.org/indepth/math*.

22. How would you recommend special educators proceed with acquiring effective interventions for students who are "non-responders" at Tiers 1-3?

A much greater level of collaboration among mathematics educators and special educators is needed in order to improve mathematics outcomes for students with disabilities. Unfortunately, there has been very little collaboration between mathematics educators and special educators historically, both in terms of practice and research. Like the field of reading, there is a history of differing philosophies about how mathematics instruction should be delivered, and there has been a lot of misunderstanding on both sides about what each really believes.

Mathematics is a complex area of knowledge. It also is a mode for communicating ideas, not unlike the areas of reading and writing. Mathematics educators are much more knowledgeable about the nature of K-12 mathematics. Special educators would do well to listen to and learn from their mathematics education colleagues. Special educators possess great skill in developing and applying specialized research-supported instructional practices for struggling learners. Mathematics educators would do well to listen to and learn from their special education colleagues.

Special education and mathematics education researchers and practitioners *must* collaborate in order for effective interventions to be developed and refined in ways that treat mathematics in appropriate ways and that are sophisticated enough to address the diverse needs of students with disabilities. This collaborative emphasis must also include teacher educators and district-level professional development specialists.

Hopefully, as mathematics RTI matures as an educational process, both from a research and a practice perspective, the level of sophistication and effectiveness with which instructional practice is envisioned and implemented will rise. The methods that educators employ for problem-solving individualized student mathematical learning needs must advance.

Presently, instructional change across the tiers relies mostly on two factors: increasing the level of intensity of instruction and increasing the duration of instruction. For students who do not respond to increased intensity and duration at Tier 3, what is left for them? Knowledge of the K-12 mathematics curriculum — its richness as a way of communicating ideas and connecting students to the world around them, the effective practices that make abstract mathematics meaningful, and ways to integrate these elements using differentiated structures to meet the needs of individual learners — must be a base of expertise for all educators who work with these students.

23. Where do students receiving special education services receive their mathematics instruction with respect to the least restrictive environment, and who would be responsible for administering that intervention?

The question regarding "where" a student who receives special education services for mathematics should receive instruction is first and foremost dependent upon what is stipulated in the student's individualized education plan. Hypothetically, instruction for the student may occur at any of the instructional tiers that are part of a school's mathematics RTI model.

The least restrictive environment is not a singular place (i.e., the general education mathematics Tier 1 classroom). The least restrictive environment is the mathematics learning environment that includes typical peers to the extent possible while providing the necessary instructional supports required by that student. So, theoretically, this could be at Tier 1, 2 or 3. Because students without disabilities are receiving instruction across tiers, students with disabilities who need more intensive instructional support actually have more opportunities to receive mathematics instruction with their peers without disabilities than perhaps they have in the past.

The persons responsible for providing instruction would be those stipulated by the student's IEP. This could be the special education teacher only, a related service provider only, or a combination of special education, related services and general educators, including content-based intervention "coaches" (e.g., reading coach/specialist; math coach/specialist).

Resources on Mathematics Disabilities

24. What other resources on mathematics disabilities do you recommend?

Here are a few that can help:

- Gersten, R., Clark, B..S. & Jordan, N.C. (2007). Screening For Mathematics Difficulties in Grades K-3. Center of Instruction. The document can be downloaded at *www.centeroninstruction.org*.
- LD Online® (2008). WETA (*www.ldonline.org/index.php*).

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Coleman, M.R., Buyesse, V., & Neitzel, J. (2006). Recognition response: An early intervening system for young children at-risk for learning disabilities. Research synthesis and recommendations. The FPG Child Development Institute at The University of North Carolina at Chapel Hill.

Fuchs, L.S. (2006). Principles of effective assessment for screening and progress monitoring. Presented at the Center on Effective Instruction Mathematics Summit, November 13, 2006. Center on Effective Instruction (www.centeroninstruction.org).

Fuchs, L.A., Compton, D.L., Fuchs, D., Paulsen, K., Bryant,, J., & Hamlett, C.L. (2005). Responsiveness to intervention: Preventing and identifying mathematics disability. *Teaching Exceptional Children, 37*(4), 60-63.

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Gersten, R. and Chard, D.J. (1999). Number sense. Rethinking arithmetic instruction for students with mathematical disabilities. The Journal of Special Education, 44, 18-28.

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The Final Report of the National Advisory Panel (2008). US Department of Education. Contract No. EDU04CO0015/0006.

Using Curriculum Based Measurement for Progress Monitoring in Math (2007). Ideas That Work, US Office of Special Education Programs (*www.studentprogress.org*).

Witzel, B., Ferguson, C.J., and Brown, D.S. (2007). Developing early number sense for students with disabilities. *LD OnLine Exclusive. WETA* (*www.ldonline.org/article/14618*).